Micro 2 -Problem Set 9

Question 1:

Consider the following production economy: the firms' production functions are $f_1(l_1, k_1) = l_1^a k_1^{1-a}$ and $f_2(l_2, k_2) = b \cdot \left(l_2^a k_2^{1-a}\right)$ where $0 < a < 1$.
The representative consumer's utility function is $U(x, y) = c \cdot x + y$ and the resource constraints are $k_1 + k_2 = 50$ and $l_1 + l_2 = 50$.

(1) (a) find the optimal allocation as a function of c, a, and b.

(b) identify a specific property of the optimal solution and provide an explanation for your answer.

(2) assume that $a = 1/2$, $b = 2$ and $c = 4$

(a) find the optimal allocation of resources and the optimal outcome of product X and Y.

(b) use your answer in clause (a) to derive the equilibrium prices and quantitites.

Question 2:

(1) Consider the following partial equilibrium model. The representative consumer's utility function is $U(x) = 10 \cdot \sqrt{x} + y$, and the cost function of the firm that produces X is $C(x) = x$.

find the equilibrium price and the quantitiy in the market.

(2) Consider the following production economy: the firms' production functions are $f_1(l_1, k_1) = l_1^{1/2} k_1^{1/2}$ and $f_2(l_2, k_2) = l_2^{1/2} k_2^{1/2}$.
The representative consumer's utility function is $U(x, y) = 10 \cdot \sqrt{x} + y$ and the resource constraints are $k_1 + k_2 = 50$ and $l_1 + l_2 = 50$.

(a) find the optimal allocation of resources and the optimal outcome of product X and Y.

(b) use your answer in clause (a) to derive the equilibrium prices and quantitites.

(3) compare the answers of clause (a) and clause (b) and provide an economic explanation for your result.

Question 3:

Consider the following production economy: the firms' production functions are $f_1(k_1, l_1) = \frac{1}{2}\left(\sqrt{l_1} + \sqrt{k_1}\right)$ and $f_2(k_2, l_2) = \frac{1}{2}\left(\sqrt{l_2} + \sqrt{k_2}\right)$
The representative consumer's utility function is $U(x, y) = x^{1/2} y^{1/2}$ and the resource constraints are $k_1 + k_2 = 50$ and $l_1 + l_2 = 50$.

(a) find the efficiency locus in the economy.

(b) find the production possibility frontier.

(c) find the optimal allocation in the economy.

(d) find the equilibrium prices.

Question 4:

Consider the following production economy: the firms' production functions are $f_1(l_1, k_1) = 2 \cdot \left(l_1^{1/3} k_1^{2/3}\right)$ and $f_2(l_2, k_2) = l_2^{1/3} k_2^{2/3}$.
The representative consumer's utility function is $U(x, y) = x^{1/3} l^{2/3}$ and the resource constraints are $k_1 + k_2 = 60$ and $l_1 + l_2 = 60$.

(a) find the efficiency locus in the economy.

(b) find the production possibility frontier.

(c) find the optimal allocation in the economy.

(d) find the equilibrium prices.

(e) what will be the proportion in the optimal allocation of X and Y if the consumer's utility function is $U(x, y) = x^a y^{1-a}$ (5 points)