Micro 2 -Problem Set 8

Question 1:

Consider the following production economy: $f_1(l_1, k_1) = l_1 k_1$ and $f_2(l_2, k_2) = l_2 k_2^{1/2}$ where $\overline{L} = 20$ and $\overline{K} = 30$

(a) present the efficiency locus as the set of inputs bundles (l_1, k_1) for firm 1.

(b) Assume now that the representative consumer's utility is

$$U(x, y) = x \cdot y$$

(i) consider the allocation $(l_1, k_1) = (5, 12)$ and $(l_2, k_2) = (15, 18)$; show that it is Pareto efficient.

(ii) consider the allocation $(l_1, k_1) = (10, 18)$ and $(l_2, k_2) = (10, 12)$; show that it is not Pareto efficient.

(iii) show that the representative consumer prefers the latter allocation over the former allocation.

(iv) find an allocation on the efficiency locus that is better for the consumer than the latter allocation.

Question 2:

Consider the following production economy: the firms' production functions are $f_1(l_1, k_1) = l_1^{1/2} k_1^{1/2}$ and $f_2(l_2, k_2) = l_2^{1/2} k_2^{1/2}$.
The representative consumer's utility function is $U(x, y) = x^{1/2} y^{1/2}$ and the resource constraints are $k_1 + k_2 = \overline{K}$ and $l_1 + l_2 = \overline{L}$.

(1) (a) find the optimal allocation (i.e., the allocation of inputs among the firms and the produced bundle) as a function of $\overline{K}$ and $\overline{L}$.

(b) show that the optimal allocation is homegenous of degree 1 in $\overline{K}$ and $\overline{L}$, i.e., if we look at resources $t \cdot \overline{K}$ and $t \cdot \overline{L}$, the optimal allocation that you found in clause (a) is multiplied by t.

(c) provide an economic explanation for your result.

Question 3:

Consider the following production economy: the firms' production functions are $f_1(l_1, k_1) = l_1^a k_1^{1-a}$ and $f_2(l_2, k_2) = l_2^a k_2^{1-a}$ where $0 < a < 1$ and $0 < b < 1$
The representative consumer's utility function is $U(x, y) = x^\alpha y^{1-\alpha}$ and the resource constraints are $k_1 + k_2 = A$ and $l_1 + l_2 = A$.

Solve the optimal allocation as a function of the parameters. Show graphically using the Edgeworth box and the production possibility frontier the changes that occur in the optimal allocation of inputs and consequentially in the optimal bundle of consumption when:

(1) the parameter α increases.

(2) the parameter A increases.

provide an economic explanation to your result.