Micro 2 -Problem Set 6

Question 1:

Consider the following exchange economy: $U_1(x_1, y_1) = x_1 y_1$ and $U_2(x_2, y_2) = x_2 y_2$ where $\omega_1^X = 10$ and $\omega_1^Y = 20$ and $\omega_2^X = 20$ and $\omega_2^Y = 10$.

(a) present the contract curve as the set of bundles (x_1, y_1) for agent 1.

(b) present the set of Pareto efficient allocations that are better for both agents than the initial endowment.

(c) look at the bundle $(x_1, y_1) = (x_2, y_2) = (15, 15)$ is this bundle Pareto efficient? prove your answer and provide an economic explanation for your result.

Question 2:

Consider the following exchange economy: $U_1(x_1, y_1) = x_1^{1/3} y_1^{2/3}$ and $U_2(x_2, y_2) = x_2 y_2^{1/2}$ where $\omega_1^X = 8$ and $\omega_1^Y = 8$ and $\omega_2^X = 16$ and $\omega_2^Y = 16$.

(a) present the contract curve as the set of bundles (x_1, y_1) for agent 1.

(b) present the set of Pareto efficient allocations that are better for both agents than the initial endowment.

(c) look at the bundle $(x_1, y_1) = (x_2, y_2) = (12, 12)$ is this bundle Pareto efficient? prove your answer and provide an economic explanation for your result.

Question 3:

Consider the following exchange economy: $U_1(x_1, y_1) = x_1 y_1$ and $U_2(x_2, y_2) = x_2 y_2^{1/2}$ where $\omega_1^X = 5$ and $\omega_1^Y = 10$ and $\omega_2^X = 15$ and $\omega_2^Y = 20$.

(a) present the contract curve as the set of bundles (x_1, y_1) for agent 1.

(b) Assume now that there is a social planner whose welfare can be represented as follow

$$W((x_1, y_1), (x_2, y_2)) = U_1(x_1, y_1) \cdot U_2(x_2, y_2)$$

(i) consider the allocation $(x_1, y_1) = (5, 12)$ and $(x_2, y_2) = (15, 18)$; show that it is Pareto efficient.

(ii) consider the allocation $(x_1, y_1) = (10, 18)$ and $(x_2, y_2) = (10, 12)$; show that it is not Pareto efficient.

(iii) show that the social planner prefers the latter allocation over the former allocation.

(iv) find an allocation on the contract curve that is better for the planner than the latter allocation.

Question 4:

Consider the following exchange economy: $U_1(x_1, y_1) = x_1^{1/3} y_1^{2/3}$ and $U_2(x_2, y_2) = x_2 y_2^{1/2}$ where $\omega_1^X = 8$ and $\omega_1^Y = 8$ and $\omega_2^X = 16$ and $\omega_2^Y = 16$.

(a) find the equilibrium price ratio $\frac{p_x^*}{p_y^*}$ and the equilibrium allocation.

(b) show that for any price ratio $\frac{p_x}{p_y} < \frac{p_x^*}{p_y^*}$ there will be over demand for good X.

(c) show that for any price ratio $\frac{p_x}{p_y} > \frac{p_x^*}{p_y^*}$ there will be over demand for good Y.

Question 5:

Consider the following exchange economy $U^1(x_1, y_1) = x_1^a \cdot y_1^b$ and $U^2(x_2, y_2) = x_2^a \cdot y_2^b$ (note that both agents have the same utility function) where $\omega_1^X = 15$ and $\omega_1^Y = 15$ and $\omega_2^X = 15$ and $\omega_2^Y = 15$.

(a) assume that the ratio of the equilibrium prices is $\frac{p_x^*}{p_y^*} = 2$, find the ratio $\frac{a}{b}$.

(b) assume that the ratio of the equilibrium prices is $\frac{p_x^*}{p_y^*} = \frac{3}{2}$, find the ratio $\frac{a}{b}$.

Question 6:

Consider the following exchange economy: $U_1(x_1, y_1) = x_1 y_1$ and $U_2(x_2, y_2) = x_2 y_2^{1/2}$ where $\omega_1^X = 5$ and $\omega_1^Y = 10$ and $\omega_2^X = 15$ and $\omega_2^Y = 20$.

(a) find the equilibrium price ratio and the equilibrium allocation.

(b) assume now that there is a social planner whose welfare can be represented as follow

$$W((x_1, y_1), (x_2, y_2)) = U_1(x_1, y_1) \cdot U_2(x_2, y_2)$$

(i) find an allocation that is not Pareto efficient which the social planner prefers to the equilibrium allocation.

(ii) find another endowment allocation under which the equilirium outcome is better for the social planner than the allocation you found in clause (i).