Micro 2 -Problem Set 5

Question 1:

For each of the following statements claim whether it is true or not. If yes, prove your answer; if not, provide a counter example.

(a) consider three allocations {A,B,C} if A is Pareto efficient and B is not Pareto efficient, then A Pareto dominates B.

(b) consider two allocations {A,B} if A is Pareto efficient and B is not Pareto efficient, then A Pareto dominates B.

(c) consider two agents 1 and 2, three allocations {A,B,C}, and a social planner whose objective function is $W(K) = U_1(K) \cdot U_2(K)$, where K is an element in $\{A, B, C\}$, and where the functions $U_1(K)$ and $U_2(K)$ represent agent 1 and agent 2 utilities respectively.

If A is Pareto efficient and B is not Pareto efficient, then the social planner prefers A to B.

Question 2:

(a) Consider the following setting $U_1(x_1, y_1) = x_1 y_1$ and $U_2(x_2, y_2) = x_2 y_2$ with $\overline{X}$ and $\overline{Y}$, and the set of allocations that satisfies $U_1(x_1, y_1) = 6$. Find the Pareto efficient allocation in this set of allocations and show that it is unique.

(b) Consider the following setting $U_1(x_1, y_1) = x_1 + y_1$ and $U_2(x_2, y_2) = 2x_2 + y_2$ with $\overline{X} \geq 6$ and $\overline{Y} \geq 6$, and the set of allocations that satisfies $U_1(x_1, y_1) = 6$. Find the Pareto efficient allocation in this set of allocations and show that it is unique.

(c) Consider the following setting $U_1(x_1, y_1) = x_1 + y_1$ and $U_2(x_2, y_2) = x_2 + y_2$ with $\overline{X} \geq 6$ and $\overline{Y} \geq 6$, and the set of allocations that satisfies $U_1(x_1, y_1) = 6$. Find the set of Pareto efficient allocations in this set of allocations.

Question 3:

Consider an exchange economy and a given indifference curve of agent 1 assume that the agents' utility functions satisfy decreasing MRS. Prove the following results (without using graphical arguments).

(a)(i) the point on agent 1's indifference curve at which agent 2's utility is maximized is Pareto efficient.

(ii) if a point on agent 1's indifference curve is Pareto efficient, then it is the point that maximizes agent 2's utility among all the points on agent 1's indifference curve.

(b)(i) at a point which maximizes agent 2's utility among all the points agent 1's indifference curve, we have that $MRS_1 = MRS_2$.

(ii) A point on agent 1's indifference curve at which $MRS_1 = MRS_2$ maximizes agent 2's utility among all the point on agent 1's indifference curve.