Micro 2 -Problem Set 10

Question 1:

Factory A is producing air conditioners. Factory B that produces bottles is located next to factory A. Due to the noise from production in factory A some of the bottles in Factory B get broken, which increases the production costs of factory B.
We denote by a the quantity produced in factory A and in b the quantity produced in factory B.
The cost of production of firm A is

$$C_A(a) = a^2$$

The cost of production of firm B is

$$C_B(b, a) = b^2 + b \cdot a$$

the price of air conditioners and bottles are $p_a = 40$ and $p_b = 80$.

(a) what is the quantity of air conditioner and the quantity of bottles in a competitive equilibrium? calculate the profit of each firm.

(b) what are the quantities produced in the efficient allocation?

(c) what is the size of the pigouvian tax t for each air conditioner that would yield an efficient allocation.

Assume now that firm A needs to get a "noise permit" for each unit it produces. There exists a competitive market for these noise permits, where the price for a permit is denoted by r. Assume that in the initial endowment firm A receives ω_A and for firm B it is ω_B and we denote by $\Omega = \omega_A + \omega_B$.

(d) write the maximization problem of firm A and of firm B.

(e) find the equilibrium in the market: what is the equilibrium price r? how many units of good A and good B are produced? does the initial allocation of permits (ω_A and ω_B) changes the equilibrium quantities?

(f) calculate the smallest number of noise permit for firm A, ω_A, that one should provide to firm A that guarantees that the equilibrium allocation with noise permit Pareto dominates the allocation with no permits from clause

Question 2:

Consider the following production economy: there are two agents - Homer and Barney; there are two products: C and W. Homer's utility is

$$U_H(c_H, w_H, c_B) = 10 + c_H^{1/2} \cdot w_H^{1/2} - c_B^{2/3} \cdot w_B^{1/3}$$

and Barney's utility is

$$U_B(c_B, w_B, c_H) = 10 + c_B^{1/2} \cdot w_B^{1/2} - c_H^{2/3} \cdot w_H^{1/3}$$

.
the initial endowment is $\left(\omega_c^B, \omega_w^B\right) = (18, 20)$ and $\left(\omega_c^H, \omega_w^H\right) = (18, 20)$.

(a) find the equilibrium allocation and prices, and calculate the agents' utilities in equilibrium.

(b) consider the same environment with an initial endowment of $\left(\omega_c^B, \omega_w^B\right) = (22, 20)$ and $\left(\omega_c^H, \omega_w^H\right) = (22, 20)$ find the equilibrium allocation and prices, and calculate the agents' utilities in equilibrium.

(c) in which economy the agents situation is better? how can you explain your answer given that each agent's utility is increasing in the consumption of each of the goods.

(d) consider the initial economy find a price other than the equilibrium price under which the allocation the agents would choose would Pareto dominate the equilibrium allocation

Question 3:

Consider an economy with N agnets and two goods, a public good G and money Y. The utility function of agent i is $U_i = V(g) + y_i$ (note that all agents have the same valuation function of the public good). All agents have the same income I (assume that it large enough so it never binds the agent's ablitity to purchase more g. Assume that the cost of one unit of the public good is 1 dollar.

(a) show that for any N there exists a Nash equilibrium where all agents take the same action (symmetric equilibrium) prove that it is a Nash equilibrium. How the overall quantity of the public good in equlibrium changes as the number of agents in the economy N changes?

(b) derive what is the quantity that maximizes the social welfare? how this quantity changes as a function of N? what does this tell us about how the size of the market failure changes as the number of agents in the economy grows?

Question 4:

Consider the public good model we considered in class and assume that the agents are altruists, i.e., they internalize to some extent the other agent utility?

assume that the agents are altruistic, specifically:

the utility function of agent 1 equals $V_1(g_1+g_2)+y_1+\alpha\cdot(V_2(g_1+g_2)+y_2)$ with $\alpha \leq 1$
the utility function of agent 2 equals $V_2(g_1+g_2)+y_2+\beta\cdot(V_1(g_1+g_2)+y_1)$ with $\beta \leq 1$

derive what would be the possible equilibria in this case; show that the equilibrium quantity of the public good is greater than the case where the agents are selfish, that it increases when α and β increase; and that when either $\alpha = 1$ or $\beta = 1$ the equilibrium quantity is equal to the socially optimal quantity.