Micro 2 -Problem Set 1

Question 1:

Consider a firm with the following production function $f(l) = \sqrt{l}$. Assume that the firm takes the price for its good as given, i.e, $R(q) = p \cdot q$ where p is the price for the good and q is the quantity of the good.

1. Find the optimal quantity, i.e., the quantity that maximizes the firm profit, as a function of the price for the good p and of the wage of labor w.
2. how the optimal quantity changes as a function of p and w?
3. how the firm's optimal profit changes as a function of p and w?

Question 2:

Consider a firm that produces using a single input - labor, and whose production function is denoted by $f(l)$. Moreover assume that the firm is a price taker, i.e, that face fixed prices for its output, denoted by p, and for the labor it uses, w. Define the firm's optimal profit function Π^* to be the profit that it makes when it chooses its quantity to be q^* - the quantity that maximizes its profit. Mathematically $\Pi^*(w, p) = p \cdot q^* - C(q^*)$, where $c(q^*) = w \cdot f^{-1}(q^*)$ and q^* is the quantity that the firm chooses to maximize its profit. Explain why the following properties of the optimal profit function are true:

1. The optimal profit function is homogenous of a degree 1. That is if we look at prices p and w and multiply them by some positive number $t > 0$ then the optimal profit is also multiplied by t.

2. The optimal profit function is increasing in p.

3. The optimal profit function is decreasing in w.

Question 3:

Answer whether the following claims are true or false. If a claim is true, provide a logical argument that proves it. If it is not true, find an example that shows the opposite:

1. Assume that the derivative of the profit function is equal to zero for a unique q^* and assume that the firm's marginal cost $C(q)$ is a decreasing function, then the firm's optimal quantity is not q^*.

2. Assume that $\Pi(q) = p \cdot q - C(q)$. Assume that the derivative of the profit function is equal to zero for a unique q^* and assume that the firm's marginal cost $C(q)$ is a decreasing function, then the firm's optimal quantity is not q^*.

3. If the firm's profit function is a convex function, i.e., $\Pi''(q) > 0$, then the firm either does not produce or it produces as much quantity as it can possibly can.

4. Assume that $\Pi(q) = p \cdot q - C(q)$. If the marginal cost of the first unit is greater than the price, i.e., $C'(0) > p$, then the firm chooses not to produce.